Optimal Design and Management of a Smart Residential PV and Energy Storage System

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Abstract— **Solar photovoltaic (PV) technology has been widely deployed in large power plants operated by utility companies. However, the home owners are not yet convinced of the saving cost benefits of this technology, and consequently, in spite of government subsidies, they have been reluctant to install PV systems in their homes. The main reason for this is the absence of a complete and truthful analysis which could explain to home owners under what conditions spending money on a PV system can actually save them money over a long-term, but known, time horizon. This paper thus presents a design and management mechanism for a smart residential energy system comprising PV modules, electrical energy storage banks, and conversion circuits connected to the power grid. First, we figure out how much savings can be achieved by a system with given PV modules and EES bank capacities by optimally solving the daily energy flow control problem of such a system. Based on the daily optimization results, we come up with the optimal system specifications with a fixed budget. Experiments are conducted for various electricity prices and different profiles of PV output power and load demand. Results show that the designed system breaks even in 6 years and in the system lifetime achieves up to 8% annual profit besides paying back the budget.**

I. Introduction

Electrical energy consumption generally ramps up significantly during certain hours of a day (a.k.a., peak hours). On the other hand, the amount of electricity generated by the utility companies in a given area is relatively constant (with only small fluctuations around a mean value). Consequently, to avoid blackouts, utility companies must overprovision their power generation capacity so that they can meet the peak demand of the users in their service area. This means a lot of the generated power is wasted during off-peak hours (sometimes called, base-load power hours) [10].

To incentivize residential users to reduce their peak hour energy consumption, utility companies have begun to employ a *time-of-day pricing policy* with higher electric energy prices during peak hours. Given this pricing scheme, one way for residential users to lower their electricity bills is to perform *load shifting* whereby users' high energy-consuming but non-critical tasks are transferred from peak to off-peak hours [12][16]. In practice, this method has limited effectiveness because only a small fraction of tasks is transferrable. Another method, which deploys grid-connected *electrical energy storage* (EES) systems for households, stores some electrical energy during off-peak hours and uses the stored energy during peak hours. As a result, energy is bought at a lower price during off-peak hours, stored, and consumed during peak hours to avoid paying higher energy prices during those hours. For example, Zhu *et al.* [17] present a framework to design and control a residential energy storage system, combining different types of EES banks to minimize a user's electric bill.

Yet another method is to utilize local power generation capacity in homes, so that the homeowners can use (store for future use) the generated power by these local renewable power sources during the peak (off-peak) hours. These renewable sources include wind power, solar power, geothermal, etc. Among all of these the *photovoltaic* (PV) technology has proven to be effective, relatively easily deployable, and offering good cost-performance tradeoff [2]. To investigate the potential of PV technology in reducing electricity bills and making profits for household systems, Lesourd in [9] elaborately analyzes several aspects of the economics of a grid-connected PV system, such as PV module capital cost, lifetime, electricity price, etc. Kirkegaard *et al.* [8] study the profitability of the state-of-the-art PV industry and propose several suggestions to help avoid possible trade and investment barriers. These research reports lead to a general conclusion that the PV systems are promising in making profits for residential usage, but a closer and deeper look into the characteristics of the PV power generation and load power consumption is mandatory to convince consumers of the profitability.

With PV modules installed, residential users can reduce electricity bills by directly supplying power from the PV modules to the load. However, there is a mismatch between the actual peak hours of PV power generation and the peak demand hours of the residential load. Figure 1 shows the solar power supply and the load demand profiles in one day: the PV power generation reaches its peak from 11:00 to 15:00 whereas the demand peak is around 18:00 in the evening. To fully utilize the harvested solar energy, PV systems necessitate built-in EES banks to store the excessive energy for later use. In turn, the performance of EES banks has a significant influence on the system efficiency and profitability.

Quite a few control algorithms and management policies have been developed for such a residential PV and energy storage system. Ha Pham *et al.* propose a reactive energy management policy for a household PV and energy storage system based on mixed integer linear programming (MILP) algorithm [6]. Riffonneau *et al.* present a predictive control system to perform peak shaving and reduce daily electricity bills, taking into account battery aging effect [13]. Wang *et al* propose a hierarchical algorithm for predicting PV generation and load consumption, and controlling energy storage systems [14]. These work mainly focus on develop-

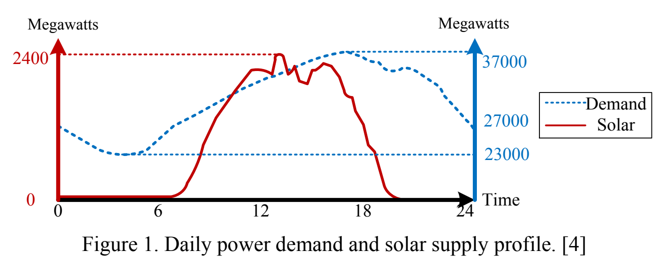

Figure 1. Daily power demand and solar supply profile. [4]

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ing an optimal control algorithm for daily energy flow to maximize saving in daily electric bill, with given PV and storage system specifications. They typically use simplified modeling of EES modules assuming 100% charging and discharging efficiency or uniform capacity degradation (aging) speed, and therefore may result in misleading conclusions when detailed characteristics of EES modules are omitted.

This paper proposes a framework for the optimal design and management of residential PV and energy storage systems. We first develop a daily energy flow control policy, which manages the energy flow between the load device and the three sources of energy, namely the PV modules, the EES bank, and the grid. We show the results on daily energy cost reductions with different capacities of the PV modules and the EES bank in the system. Based on this optimal control policy, we provide a design methodology to determine the system specifications, namely the capacity of PV modules and the type, capacity, and usage limit of the EES banks with a given budget. The major contributions of this paper include:

- Provided a daily energy flow management policy based on a detailed analysis of the features of EES elements, taking into account the system's long-term performance;
- Proposed a design methodology of choosing the best residential PV and energy storage system specifications, including the capacities of PV modules and the types and specifications of EES banks, with given budgets;
- Presented a case study, which shows the break-even time results as well as the annual profits of the designed system with respect to different budgets, different peak hour prices and different types of EES banks.

The rest of the paper is organized as follows. Section II elaborates the models of PV modules, EES elements, the electricity pricing policy, and the entire PV and storage system. Section III and IV present the daily energy flow control problem and the global design optimization problem, respectively. Section V provides the simulation results and the paper is concluded in Section VI.

II. System Model

A. PV Modules

Despite the rapid maturation of the PV technology, PV modules are still quite expensive. The cost of a PV system includes system design and installation, insurance, inverter maintenance, taxes, etc. The unit price of the state-of-the-art commercialized PV system is about \$5 per watt generated at the maximum power point (MPP), accounting for all the above-mentioned cost components [2].

The long-term performance of PV modules is far more stable than most of the traditional electrical systems. The average annual degradation rates of power generation of a PV module, including all its internal circuit breakage cases, is below 1% [2][6]. Moreover, the expected lifetime of PV modules can be 30 years or longer [2]. Unlike batteries, PV modules experience gradual aging process instead of sudden performance degradation. PV modules can actually operate so long as the output power still satisfies the demands.

In conclusion, although the initial purchase cost of PV modules is typically high, it is promising to apply them to households for-long term electrical energy saving.

B. EES Elements

Common commercial EES elements include lead-acid batteries, Lithium-ion (Li-ion) batteries, metal-air batteries NiMH batteries, and supercapacitors. Table 1 shows the comparison of primary performance metrics of the above EES elements, where the strengths and weaknesses of each type are highlighted in **boldface** and *italic*, respectively.

TABLE 1. PERFORMANCE METRICS COMPARISON OF EES ELEMENTS.

	Capital cost (\$/kWh)	Cycle efficiency	Cycle life	Self-discharge per day
Lead-acid	**100-200**	70-90%	*500-800*	0.1-0.3%
NiMH	450-1,000	66%	500-1k	0.5-1%
Li-ion	*600-2,500*	**>90%**	**1k-10k+**	0.1-0.3%
Metal-Air	**10-60**	*<50%*	100-300	Very small
Supercap	*20k-50k*	**>90%**	**50k+**	*20-40%*

Among the various types of EES elements, we need to select the most appropriate one for the residential PV and energy storage system. In the first place, we must consider the capital cost of the selected EES element while satisfying the energy capacity requirement of the residential user. The capital cost is represented in the forms of *unit price*, defined by the dollar cost per unit of stored energy (\$/kWh). Lead-acid batteries and metal-air batteries achieve much lower unit price compared to Li-ion batteries and NiMH batteries. Supercapacitors, on the other hand, have unacceptably high unit price for home users.

The performance of the residential PV and energy storage system is largely dependent on the *discharge efficiency* of the EES bank, which is defined as the ratio of EES bank output power to the actual degradation rate of its stored energy. The discharge efficiency of batteries is largely affected by *rate capacity effect*, i.e., the actual charge loss rate I_{eq} inside a battery is a superlinear function of its discharge current I_{dis}:

$$I_{eq} = \left(\frac{I_{dis}}{I_{ref}}\right)^k I_{ref} \qquad (1)$$

where I_{ref} is the reference discharge current. It is typically $Q/20$ where Q is the *nominal full-charge capacity* (in Ah) of a battery, meaning that it takes 20 hours to use I_{ref} to fully discharge the battery. The exponent k is a constant greater than 1, which reflects the significance of rate capacity effect. For lead-acid batteries, metal-air batteries, and NiMH batteries, the value of k (more than 1.3) is much higher than that of Li-ion batteries (less than 1.1).

While improving the daily performance of the PV and energy storage system, we also need to consider the *cycle life* of EES elements. The cycle life is defined as the number of full charge/discharge cycles an EES element can perform before its capacity drops to a specific percentage threshold (typically 60% to 80%) of its initial *full-charge capacity* (FCC). Li-ion batteries and supercapacitors significantly outperform other types of EES elements in terms of cycle life.

Taking all the above-mentioned performance metrics into consideration, we select lead-acid batteries and Li-ion batteries and analyze the profitability of two PV and energy storage systems in this paper, one with lead-acid batteries as the EES bank and the other with Li-ion batteries.

C. Electricity Pricing Policies

To reflect the variations in electricity generation cost and consumption demand during different hours of a day, utility companies employ *time-of-day pricing policy*, with high unit energy price during the peak hours and low unit energy price during off-peak hours. In addition, as cooling requirements ramp up in summer daytime and exert more pressure on the grid, utility companies usually set higher peak hour prices for high seasons (e.g. summer). A typical electricity pricing policy [5] is provided in Table 2. We use u_{PK}, u_{BS} to indicate the peak-hour and base-hour electricity prices hereinafter (BS stands for base-load power hours). The peak-to-base ratio of electricity price, given by $\alpha = u_{PK}/u_{BS}$,

TABLE 2. TIME-OF-DAY PRICING POLICY.

	u_{PK}		u_{BS}
	High season	*Low season*	
Electricity price (\$/kWh)	0.3027	0.1098	0.0116

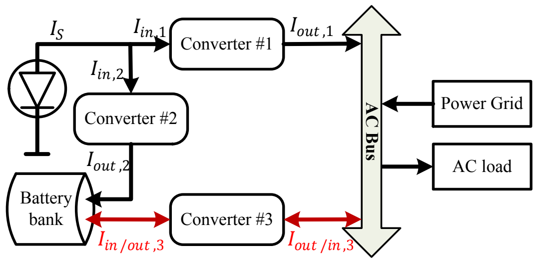

Figure 2. Residential PV and energy storage system structure.

has high impact on the daily storage management policy. These impacts are demonstrated in Section V.B.d).

D. Residential PV and Energy Storage System

Figure 2 shows the structure of the proposed residential PV system with energy storage. Three converters connect the PV modules, the EES bank, and the AC Bus. These converters control the energy flow by regulating their input or output currents: Converter #1 is a DC-AC inverter. It is turned ON as long as the PV output power is non-zero (i.e., during daytime.) The input and output currents of Converter #1 are determined by the PV power profile and the load demand profile. Converter #2 is a unidirectional DC-DC converter. It is ON only if the PV modules have excessive power to charge the EES bank after supplying power to the load. Converter #3 is bidirectional. It contains a DC-AC inverter and an AC-DC rectifier, supporting two-way energy flow between the EES bank and the AC bus. When the EES bank is providing power to the load, the DC-AC inverter is ON (current flows from left to right along the red arrows in Figure 2); and when the EES bank gets charged by the grid, the AC-DC rectifier is ON (right to left). We use η_1, η_2, and η_3 to denote the efficiencies of DC-AC inverter, DC-DC converter, and AC-DC rectifier, respectively.

The system operates in the following way: If the PV modules energy generation is sufficient to supply the load demand, the surplus energy is stored in the EES bank. If the load demand cannot be satisfied by the PV modules, the situations are different for base hours and peak hours: In base hours, the grid provides energy to both the remaining load demand and the EES bank for storage. In peak hours, the EES bank together with the PV modules supply the load, and if still not sufficient, the load gets energy from the grid.

III. DAILY ENERGY FLOW CONTROL

Before making decisions on the system design, we first investigate how much energy cost a home user could actually save when equipped with PV modules and the EES bank with different capacities. This section defines the problem of daily energy flow control with a given PV and energy storage system specification and provides the problem solution, and then proposes a lifetime-aware daily control problem based on the solution of daily optimization.

A. Daily Energy Flow Control Problem

The basic idea of daily energy flow control is two-fold: (i) to shift the peak output power of PV modules for better load peak shaving and (ii) to store energy (in the storage) during base hours for future use in high-price hours. A good energy flow control policy should make effective use of the PV power generation and the EES bank capacity, while minimizing electricity bills and energy loss during system operation. We assume that PV power generation profile and load demand profile are given at the beginning of the day by effective prediction algorithms such as in [14], to emphasize on figuring out an optimal energy flow control policy of the system.

We use N decision epochs (hence N time slots) per hour in the proposed energy flow control method. The index set of decision epochs during peak hours is denoted by $PK = \{t_1N+1, t_1N+2, \ldots, t_2N\}$, and the index set of base hours is denoted by $BS = \{1,2,\ldots,t_1N\} \cup \{t_2N+1, t_2N+2, \ldots, 24N\}$, where t_1 and t_2 are the start and end time (in hours) of peak hours. At the i-th decision epoch, the load energy demand is given by E_i^{load}, and the PV module power supply is given by $Q_{PV} \cdot p_i$, which is the multiply of the PV module capacity Q_{PV} (in W) and the power supply p_i of a unit-size PV module.

The daily control policy needs to determine the values of the charge/discharge currents of the EES bank to control the energy flow. Let $x_{t_1N+1}, \ldots, x_{t_2N}$ denote the EES bank currents in the corresponding time slots of peak hours. $x_i \geq 0$ means that the EES bank is getting discharged, and $x_i < 0$ means getting charged. The goal is to maximize the daily energy cost saving, which is the subtraction of original energy cost without the PV and energy storage system by the new daily cost with such a system installed. We need to consider the following two components to calculate the daily energy cost saving:

1) The load energy cost reduction, given by

$$\sum_{i\in PK} u_{PK} \cdot \Delta E_i^{load} + \sum_{i\in BS} u_{BS} \cdot \Delta E_i^{load} \tag{2}$$

where ΔE_i^{load} is the amount of energy provided to the load by the PV and energy storage system. As stated in Section II.D, there are two cases: (i) when PV output power has extra power besides supporting the load, i.e. $E_i^{load} < \frac{\eta_1 p_i Q_{PV}}{N}$, the load demand is fully supplied by the PV output power, i.e., $\Delta E_i^{load} = E_i^{load}$, and the EES bank is getting charged, i.e., $x_i < 0$; (ii) when the PV output power is not enough to supply the load power demand itself, i.e., $E_i^{load} \geq \frac{\eta_1 p_i Q_{PV}}{N}$, the EES bank gets charged by the grid in base hours, and provides energy to the load in peak hours (and the EES bank is getting discharged in this case, i.e., $x_i \geq 0$). The load energy reduction ΔE_i^{load} is thereby calculated by

$$\Delta E_i^{load} = \begin{cases} \frac{\eta_1}{N}(x_i V_B + p_i Q_{PV}), \text{if } E_i^{load} \geq \frac{\eta_1 p_i Q_{PV}}{N} \text{ and } i \in PK \\ \eta_1 \, p_i Q_{PV}/N, \quad \text{if } E_i^{load} \geq \frac{\eta_1 p_i Q_{PV}}{N} \text{ and } i \in BS \\ E_i^{load}, \quad \text{if } E_i^{load} < \frac{\eta_1 p_i Q_{PV}}{N} \end{cases} \tag{3}$$

where V_B is the terminal voltage of the battery bank.

2) The additional energy cost from charging the EES bank during base hours:

$$u_{BS} \cdot \frac{q_{ini} V_B - \eta_2 \sum_{i\in BS} \mathcal{E}_i}{\eta_3} \tag{4}$$

where q_{ini} is the initial charge stored in the EES bank at the beginning of peak hours, and $\mathcal{E}_i$ is the excessive energy of PV modules during the i-th time slot of base hours, calculated by

$$\mathcal{E}_i = \begin{cases} 0, & \text{if } E_i^{load} \geq \frac{\eta_1 p_i Q_{PV}}{N} \\ \frac{p_i Q_{PV}}{N} - \frac{E_i^{load}}{\eta_1}, & \text{if } E_i^{load} < \frac{\eta_1 p_i Q_{PV}}{N} \end{cases}, i \in BS \tag{5}$$

While maximizing the objective function, variables (current values $x_{t_1N+1}, \dots, x_{t_2N}$ and the initial charge q_{ini}) must satisfy the following constraints:

1) The positive or negative signs of $x_{t_1N+1}, \dots, x_{t_2N}$, indicating the EES bank is getting discharged or charged, satisfy:

$$\forall i \in PK, \begin{cases} x_i \geq 0, \text{if } E_i^{load} \geq \dfrac{\eta_1 p_i Q_{PV}}{N} \\ x_i < 0, \text{if } E_i^{load} < \dfrac{\eta_1 p_i Q_{PV}}{N} \end{cases} \quad (6)$$

2) The charge currents constraints: When the EES bank is getting charged by the PV modules, the input energy of storage cannot exceed the surplus energy $\frac{p_i Q_{PV}}{N} - \frac{E_i^{load}}{\eta_1}$ of PV besides supplying the load, i.e.,

$$\forall i \in PK, -\frac{x_i V_B}{N} \leq \eta_2 \left(\frac{p_i Q_{PV}}{N} - \frac{E_i^{load}}{\eta_1} \right), \text{if } x_i < 0 \quad (7)$$

3) The load energy constraints, meaning that the energy provided to the load cannot exceed the load demand (i.e., selling electricity back to the grid is not allowed):

$$E_i^{load} - \Delta E_i^{load} \geq 0, \forall i \in PK \quad (8)$$

4) The battery capacity constraints: the remaining charge of the EES bank during any time slot should neither fall below zero nor exceed the capacity Q_B:

$$\begin{gathered} Q_B \geq q_{ini} - \sum_{i=t_1N+1}^{j} \Delta Q_i \geq 0, \forall j \in PK \\ Q_B \geq q_{ini} \geq 0 \end{gathered} \quad (9)$$

where ΔQ_i is the amount of charge loss during the i-th time slot in the EES bank. Similar to the discharge current x_i, ΔQ_i is positive if the EES bank is being discharged, and negative otherwise. As mentioned above, batteries experience rate capacity effect during discharging, and hence, the charge-/discharge rates ΔQ_i is calculated by

$$\Delta Q_i = \begin{cases} \dfrac{x_i}{N}, & \text{if } E_i^{load} < \dfrac{\eta_1 p_i Q_{PV}}{N} \\ \left(\dfrac{20 x_i}{Q_B} \right)^k \dfrac{Q_B}{20} / N, & \text{if } E_i^{load} \geq \dfrac{\eta_1 p_i Q_{PV}}{N} \end{cases} \quad (10)$$

where $\frac{Q_B}{20}$ is the reference current.

With the analysis above, we formulate the daily energy flow control problem as follows:

Given:

1) The capacity of PV modules Q_{PV} (in W) and EES bank capacity Q_B (in Ah);
2) The terminal voltage of the EES bank V_B;
3) The base hour and peak hour unit energy price u_{BS}, u_{PK};
4) Batteries' rate capacity effect coefficient k;
5) Residential load energy profile $E_i^{load}, i = 1, \dots, 24N$;
6) PV output power (per unit capacity) profile $p_i, i = 1, \dots, 24N$;
7) The power conversion efficiencies of DC-AC inverters, DC-DC converters, and AC-DC rectifiers: η_1, η_2, and η_3, respectively.

Find: Charge/Discharge currents $x_{t_1N+1}, \dots, x_{t_2N}$ of the EES bank during peak hours, and the initial charge q_{ini}.

Maximize: The daily energy cost reduction:

$$\begin{aligned} R\left(x_{t_1N+1}, \dots, x_{t_2N}, q_{ini}\right) &= \sum_{i \in PK} u_{PK} \cdot \Delta E_i^{load} \\ &+ \sum_{i \in BS} u_{BS} \cdot \Delta E_i^{load} - u_{BS} \cdot \frac{q_{ini} V_B - \eta_2 \sum_{i \in BS} \mathcal{E}_i}{\eta_3} \end{aligned} \quad (11)$$

where ΔE_i^{load} , $\mathcal{E}_i$ is given by (3) and (5), respectively.

Subject to:

1) The positive or negative signs of currents, given by (6);
2) The charge current constraints, given by (7);
3) The load energy constraints, given by (8);
4) The battery capacity constraints, given by (9).

The daily control problem can be transformed to a convex optimization problem if we treat the set of ΔQ_i as optimization variables instead of the set of x_i. The battery capacity constraints in (9) become linear constraints while every constraint in Equation (6) to (8) can be converted into the form of $\Delta Q_i \leq c$ or $\Delta Q_i \geq c$ where c is a constant. In this way, all of the constraints are linear constraints. Maximizing the objective function which is concave is equivalent to minimizing a convex objective function. Therefore, the daily energy flow control problem can be solved by standard convex optimization tools within polynomial time.

The maximum energy cost reduction of the d-th day is expressed as a function $f_d(Q_{PV}, Q_B)$ of system PV capacity Q_{PV} and battery capacity Q_B. We sum up these daily optimization results to obtain the maximum seasonal energy cost reductions $F_{LS}(Q_{PV}, Q_B)$ for low season and $F_{HS}(Q_{PV}, Q_B)$ for high season. They are stored in two-dimensional look-up tables (LUTs).

B. Daily Battery Usage Control Problem

The optimal solution in the daily flow control problem is allowed to fully charge and discharge the batteries. Utilizing the full battery capacity range means storing as much energy as possible into the battery reservoir, therefore reducing energy demand from the grid and maximizing the energy cost reduction. However, fully charging and discharging a battery results in more severe aging effect, thereby not only causing faster system performance degradation, but also shrinking the battery lifetime [1][11]. As discussed in [17], limiting a proper percentage of the entire battery capacity to be usable can extend the battery lifetime, slow down the performance degradation, and therefore achieve more profits throughout the system lifetime.

Assume the capacity limit is set to l_B ($l_B \leq 1$), meaning the usable capacity range is 0 to $l_B Q_B$. The maximum daily reduction becomes a function of three variables: $g_d(Q_{PV}, Q_B, l_B)$. If we use $f_d(Q_{PV}, l_B Q_B)$ to estimate $g_d(Q_{PV}, Q_B, l_B)$, the difference is the discharge rate ΔQ_i in Equation (10):

$$\begin{aligned} (\Delta Q_i)_f &= \left(\frac{20 x_i}{l_B Q_B} \right)^k \frac{l_B Q_B}{20} / N = \left(\frac{20 x_i}{Q_B} \right)^k \frac{Q_B}{20} / N \cdot l_B^{1-k} \\ &\geq \left(\frac{20 x_i}{Q_B} \right)^k \frac{Q_B}{20} / N = (\Delta Q_i)_g \ (l_B < 1 \text{ and } k > 1) \end{aligned} \quad (12)$$

where $(\Delta Q_i)_f$, $(\Delta Q_i)_g$ are the values of ΔQ_i for $f_d(Q_{PV}, l_B Q_B)$ and $g_d(Q_{PV}, Q_B, l_B)$, respectively. This means the result $f_d(Q_{PV}, l_B Q_B)$ overestimates the discharge currents, hence underestimating the usable battery capacity as well as the overall energy cost savings, i.e., $f_d(Q_{PV}, l_B Q_B) \leq g_d(Q_{PV}, Q_B, l_B)$. Our experiments show that this error is within 4%.

IV. GLOBAL SYSTEM DESIGN

A. Battery Capacity Degradation

The effective FCC of a battery gradually drops cycle by cycle. For a lead-acid battery, we assume constant degradation rate before it reaches end-of-life. For a Li-ion battery, we adopt the aging model described in [15]. The capacity degradation of a Li-ion battery is a function of the number of finished charge/discharge cycles and the capacity range in use. The number of full cycles for a certain day is calculated by

$$C = \frac{1}{2Q_B} \left(\sum_{i \in PK} |\Delta Q_i| + q_{ini} \right) \quad (13)$$

Similar to the daily cost reduction results, C is also a function of Q_{PV}, Q_B. Simulation results show that there is a big difference between the numbers of cycles in the days of high season and low season. Therefore, we average the C values in low season and high season separately and store the results $C_{LS}(Q_{PV}, Q_B)$, $C_{HS}(Q_{PV}, Q_B)$ in two LUTs.

The aging model of Li-ion batteries in [15] gives overestimation of the performance when the capacity limit is low (e.g. $l_B < 0.5$) because it ignores an important factor, namely the *calendar life* of Li-ion batteries. It describes the battery capacity degradation as a result of the passage of time [2]. Therefore, the actual Li-ion battery FCC degradation percentage is the maximum of the two values: calendar life degradation predicted by [2] and the result predicted by [15].

B. Global Design Problem

a) Real-Life Factors

The *time value of money* is reflected by the *discount rate* γ ($\gamma < 1$), indicating that the amount of money P of the next year is worth the same value as γP of today. The proposed PV and energy storage system can serve as long as 30 years, making it necessary to account for the time value of money while calculating the profits. Let P_j denote the annual profit of the j-th year. The amortized annual profit A is calculated by assuming the same amount of profit is made each year within the system lifetime:

$$\sum_{j=1}^{30} \gamma^{j-1} A = \sum_{j=1}^{30} \gamma^{j-1} P_j \tag{14}$$

Another influential real-life factor is the system maintenance cost. Although the capital cost of PV modules ($5/kW) has taken into account the maintenance cost of the PV modules, that of the EES bank has not been considered. The lifetime of lead-acid batteries is usually around 4 years and Li-ion batteries up to 10 years. If expecting the PV and energy storage system to serve 30 years or more, we need to replace the EES bank when the old one reaches its end-of-life. The maintenance fee occurs at times of EES bank replacement, including both the installation fee and the capital cost of EES elements.

b) Problem Formulation

With the above-mentioned factors taken into consideration, we formulate the global system design problem as follows:

Given:

1) LUTs of daily energy cost reduction in low season and high season: $F_{LS}(Q_{PV}, Q_B)$, $F_{HS}(Q_{PV}, Q_B)$;
2) LUTs of the number of cycles per day in low season and high season: $C_{LS}(Q_{PV}, Q_B)$, $C_{HS}(Q_{PV}, Q_B)$;
3) Unit price of PV modules and the EES bank: p_{PV}, p_B;
4) One-time installation fee M and discount factor γ;
5) Budget B of initial investment.

Find: PV modules' capacity Q_{PV}, EES bank capacity Q_B, and battery capacity limits l_{BH} and l_{BL} of high season and low season, respectively.

Maximize: amortized annual profit $A(Q_{PV}, Q_B, l_{BH}, l_{BL})$;
or **Minimize:** the break-even time $T_{be}(Q_{PV}, Q_B, l_{BH}, l_{BL})$.

Subject to: Budget constraint: $Q_{PV} p_{PV} + Q_B p_B + M \le B$.

The objective function can be either the annual profit or the break-even time. Since consumers are usually more interested in the actual profit the PV and energy storage system makes, we use annual profit as the objective function, but also report the break-even time of the optimized result as a reference.

We use a searched-based algorithm to solve the above problem. Since this calculation only occurs once for a user (only at the initial design of the PV and energy storage system), the timing complexity is acceptable. To evaluate each set of $(Q_{PV}, Q_B, l_{BH}, l_{BL})$, we first set the initial profit to be $P_0 = -(Q_{PV} p_{PV} + Q_B p_B + M)$. Second, we iterate year by year until we reach the system's designed lifetime (usually 30 years, the PV modules' lifetime). During this process, if the EES bank reaches its end-of-life in the j-th year, the maintenance fee $Q_B p_B + M$ will be subtracted from P_j. Finally, we amortize the total profit accumulated over 30 years by Equation (14) to get the amortized annual profit A.

V. Simulation Results

A. Daily Control Results

Figure 3 shows the daily control results of both low season and high season in California for two PV and energy storage systems, one with a lead-acid battery bank and the other with a Li-ion battery bank (referred to as *PV-lead* and *PV-Li* hereinafter). Due to higher discharge efficiency, PV-Li achieves averagely 7.42% more daily energy cost reduction in low season than PV-lead, and 7.78% more in high season.

B. Global Design Results

a) Comparison between Different Types of EES banks

The results of annual profit A and break-even time T_{be} are shown in Figure 5, comparing PV-lead and PV-Li. In both annual profit and break-even time results, the PV-lead system outperforms the PV-Li system. The main reason is that although PV-Li improves the daily reduction by less than 8% compared to PV-lead, the unit price of Li-ion batteries ($560/kWh) is more than four times that of lead-acid batteries ($128/kWh), which means not only four times higher initial cost but also four times the replacement cost during the system lifetime. The PV-Li system fails to make any profit in case of $2000 budget (zero annual profit and infinite break-even time).

To explicitly show the results of using different budgets, we assume that the PV and energy storage system should use up the budget in the simulation. The PV-lead system experiences a drop in annual profit with initial budget increasing from $6000 to $7000. This is because the marginal gain in energy cost reduction decreases whilst the replacement cost of the EES bank gets higher.

b) Different PV and EES Capacities

The results of annual profit as a function of the PV module capacity are shown in Figure 6, with different budgets given to set up a PV-lead system. We assume the entire budget is used in each case. With the increase of budget, the optimal system setting requires more PV modules adopted, but the annual profit might not get improved due to increasing maintenance cost.

c) Another Case Study in Virginia

The results presented in previous subsections are based on a multi-family load profile and a PV profile both collected in California, where the sunlight is sufficient all over the year: The daily

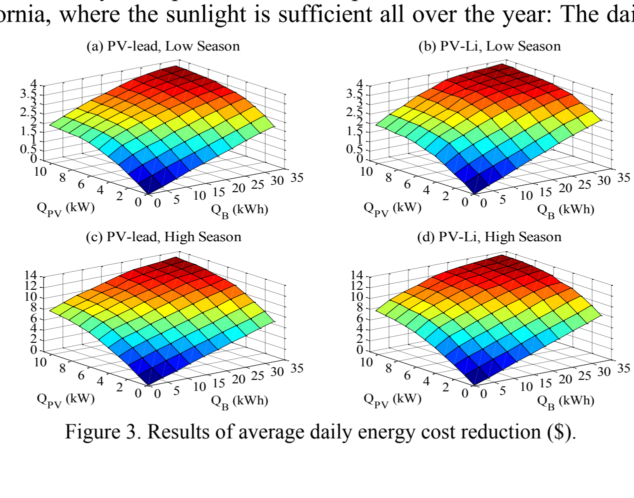

Figure 3. Results of average daily energy cost reduction ($).

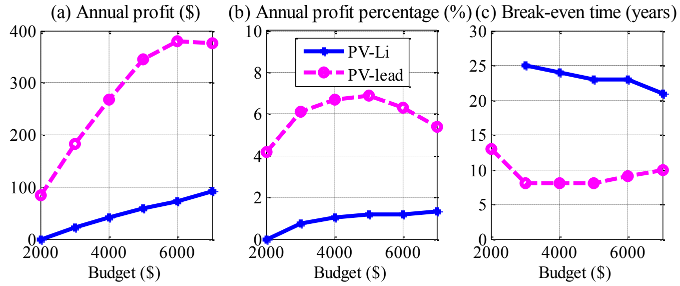

Figure 5. Comparison of the PV-lead and PV-Li systems.

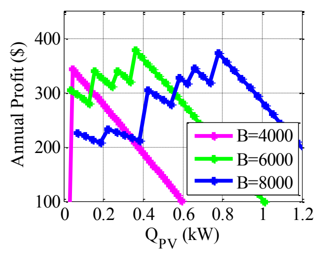

Figure 6. Different capacity settings.

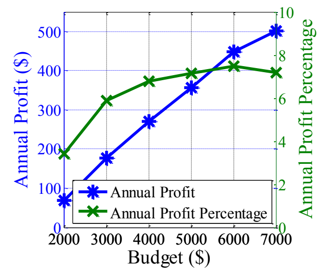

Figure 7. PV-lead in Virginia.

energy sum provided by the PV in high season is only 4.5% more than that in low season. In comparison, the state of Virginia experiences a larger seasonal difference, with the high season daily PV energy 48.2% more than the low season energy. Figure 7 shows the results of a Virginia multi-family load profile and a PV profile also collected in Virginia. The PV-lead system is able to achieve almost the same level of annual profit as in California. This is because in the low season of Virginia where the solar irradiation is low, the PV-lead system is able to get lifetime extended by using smaller percentage of the overall capacity.

d) Comparison Between Different Pricings

We discuss how the pricing variables influence the annual profits A. As mentioned in Section II.C, the peak hour energy unit price u_{PK} can be expressed by $\alpha \cdot u_{BS}$. Since the objective function R of daily flow control problem is a linear function of u_{BS}, the value of u_{BS} would not affect the daily management policy, i.e., the charge/discharge currents. However, the value of α will affect the currents during peak hours.

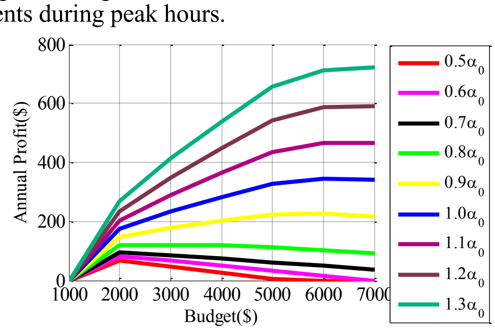

Figure 4. Annual profit results in terms of different α values.

Figure 4 shows the annual profit w.r.t. different α values, in which α_0 is the u_{PK} to u_{BS} ratio as in Table 2. We also assume the budget is fully invested in the system. It can be seen from the increasing gap between two colored curves that the annual profit increases superlinearly as the value of α increases.

VI. CONCLUSIONS

This paper presents a detailed design and management mechanism for a residential PV and energy storage system, aiming at maximizing the electricity bill savings. We first derive the formulation and optimal solution of the daily energy flow control problem to maximize the daily energy cost reduction for given PV and EES bank's capacities. Based on these results, we present a global optimization problem to determine the system specifications to maximize the amortized annual profit. The characteristics of the electrical energy storage (EES) banks, the lifetime and performance degradation of the system, as well as real-life factors such as discount rate are taken into consideration, making the optimization results more reliable. The system is tested on two PV output power and load demand profiles, one from California and the other from Virginia, as well as different electricity pricings. Simulation results show that our system achieves a break-even time of 6 years and 8% annual profit percentage besides paying back the budget.

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