Optimal Partner Selection and Power Allocation for Amplify and Forward Cooperative Diversity

Hadi Goudarzi
EE School, Sharif University of Tech.
Tehran, Iran
h_goudarzi@ee.sharif.edu

Mohamad Reza Pakravan
EE School, Sharif University of Tech.
Tehran, Iran
pakravan@sharif.edu

***Abstract*---In this paper, we present a novel algorithm for partner selection and power allocation in the Amplify-and-Forward cooperative diversity that minimizes the required total transmit power by given outage probability constraint. We represent the problem with new formulation and solve the optimal power allocation by KKT method for a fixed set of partners. For optimal partner selection, we use a novel algorithm with low complexity to find the best set with minimum required power. We present simulation results to demonstrate that the outcomes of the proposed algorithm are very close to results of full search for optimal set.**

Keywords-component; Cooperative diversity; Amplify and Forward; Constrained minimization of power.

I. INTRODUCTION

Cooperative diversity is a technique that combats the slow fading and shadowing effect in wireless communication channel [1]-[3]. In this technique, the spatially distributed users create an array of antennas to combat slow fading so the achievable rate and capacity of wireless channels will be improved saliently. The technique can also lean to reduction of the required power for transmission.

One of the most important problems in cooperative diversity is the strategy of power allocation among users [4]-[6]. Most of the related works focus on the problem to allocate a constant power to the source and its partners to achieve the minimum value of outage probability. The power allocation for the decode and forward strategy, based on simulation and observation, has been studied in [4]. Also power allocation based on the constrained optimization method has been studied in [5] and [6].

Another important challenge in cooperative diversity is to decide how many partners and which one of the many possible candidates should be chosen to cooperate with the source [7], [8]. In [7], a partner selection algorithm in an opportunistic relaying form has been proposed. It is assumed that all of the candidates of cooperation are ready to cooperate and in each packet transmission, the best partner will cooperate.

One of the recent approaches in cooperative diversity problem is minimization of power for constant rate which satisfies a constraint of outage probability or error probability [9]-[14]. In [9], the authors have expressed a short term power which is the minimum power that satisfies capacity constraint of the problem and use this for problem of constant mean of power. In [10], the authors focused on the problem of constrained minimization of power but a closed form solution was not presented. Lifetime maximization problem via cooperative nodes in wireless sensor networks is discussed in [11]. In that paper, the minimization of the total power of cooperative nodes has been studied to maximize the network life for a given error probability. In [12], the authors assume the two partners case and solve the minimization of power in the entire network. The adaptive modulation technique is applied in [13] to improve the spectral efficiency of cooperative strategy and minimize the power consumption. In [14], the authors presented two algorithms for adaptation of the number of relays for minimizing transmit power and error probability.

In this paper, we propose a novel algorithm in order to minimize the power consumption in wireless channel. This algorithm is based on AFDC. Both of the problems of partner selection and power allocation for minimizing the total power consumption with constraint of outage probability are considered in this algorithm. The results of our algorithm are very close to results of full search for optimal set. The simplicity of the proposed algorithm makes it suitable for implementation.

In section II, we express the model of wireless channel and the cooperative strategy which is employed in this paper. We express the modeling of the outage behavior with respect to partner's SNR in section III. Optimal power allocation between given set of partners is presented in section IV. In section V, Optimal partner selection problem and our novel algorithm for this selection is expressed. In section VI the results of the simulations are being expressed and we conclude this paper in section VII.

II. SYSTEM MODEL

In this paper, we assume a slow, flat fading wireless channel. In other words, the bandwidth of signal is smaller than coherence bandwidth of channel and the inverse of the rate of transmission is smaller than coherence time of channel. Noting this assumption, the fading coefficient of channel can be assumed unchanged in a few transmission periods. The large scale behavior of channel path loss is modeled with $D^{-\alpha}$ where D is the distance between transmitter and receiver and α is a positive constant between 2 and 6.

Our cooperative diversity strategy is Amplify and Forward (AF) with orthogonal transmission. In this strategy, each node selects a few partners and the partners relay the received signals from the source to the destination. Each relay can be a source in other transmission time intervals. In this paper, we

978-1-4244-2644-7/08/$25.00 ©2008 IEEE

assume that the source can select each set of the candidate partners for cooperation, i.e. it has not any limitation in the node selection process.

The partner selection and power allocation strategies of the proposed algorithm are based on the information of the means of the channel coefficients, between source and partners and between partners and destination. Also the source is not aware of the full CSI of the channels. The receiver has the information of the instantaneous CSI of the channels and uses the maximum ratio combining (MRC) to detect the source information from the signals of source and partners.

III. Modeling The Behavior Of Partners SNR In Outage Probability

To explain the behavior of the outage probability in AF strategy, we first have to explain the information term. According to [2], the source destination channel capacity in bits per time slot in AF is given by (1).

$$I = \frac{1}{m+1}\log\left(B_0 + \sum_{i=1}^{m}\frac{A_i B_i}{A_i+B_i+1}\right) \tag{1}$$

Where B_0 and B_i denote the SNR of the link between source and destination and SNR of the link between i^{th} partner and destination and A_i denotes the SNR of the link between source and i^{th} partner. Each of A_i, B_i and B_0 random variables have an exponential distribution because the amplitude of the channel coefficient has Rayleigh distribution.

To explain the behavior of (1), we must know the PDF of every term in the logarithm. In this section we want to simplify the PDF of the deficit terms in (1) to use it in our optimization algorithm. In High SNR regime, we can approximate the deficit terms by (2).

$$\frac{A_i B_i}{A_i+B_i+1} \cong \left(\frac{1}{A_i}+\frac{1}{B_i}\right)^{-1} \tag{2}$$

This approximation shows that if each mean of A_i and B_i is much greater than the other, this term can be removed from the deficit term. This shows that the PDF of the deficit term is converged to exponential distribution in two limiting cases. So, we estimate the PDF of the deficit terms by exponential distribution. We can put the mean of A_i and B_i into the deficit term to obtain the mean of exponential distribution.

$$\lambda_i = \frac{a_i b_i}{a_i+b_i+1} \tag{3}$$

Where $a_i = \frac{P_s}{d_{sr_i}^{\alpha} N_0}$ and $b_i = \frac{P_{r_i}}{d_{r_i d}^{\alpha} N_0}$ and P_s and P_{ri} denote the transmit powers of the source and i^{th} partner and d_{sri} and d_{rid} denote the distance between source and i^{th} partner and between i^{th} partner and destination and N_0 denotes the noise variance.

L shows the mean SNR of the source transmission, which is equal to $\frac{P_s}{d_{sd}^{\alpha} N_0}$. If the mean SNR of the i^{th} partner transmission will be equal to kL and if P_{r_i} has the form of (4-1), then the required amount of z_i in (4-1) must be in the form of (4-2).

$$P_{r_i} = z_i L d_{r_i d}^{\alpha} \tag{4-1}$$

$$z_i(k, D_{sr_i}) = \frac{k(1+{}^{1}/_{L} {D_{sr_i}}^{\alpha})}{1-k{D_{sr_i}}^{\alpha}} \tag{4-2}$$

D denotes the distance and is normalized in terms of d_{sd}. In high SNR, $1/L$ has a weak impact on z_i and can be ignored in z_i. For example for $k=1$ (equal SNR case), the required amount of z_i is plotted for different D_{sr_i}. We note that if d_{sr_i} is greater than d_{sd} (or $D_{sr_i} > 1$), the partner can not produce equal SNR to source. So, all of the D_{sr_i}'s in this figure are smaller than 1. The value of the dispersion parameter α is set to 2 in these figures.

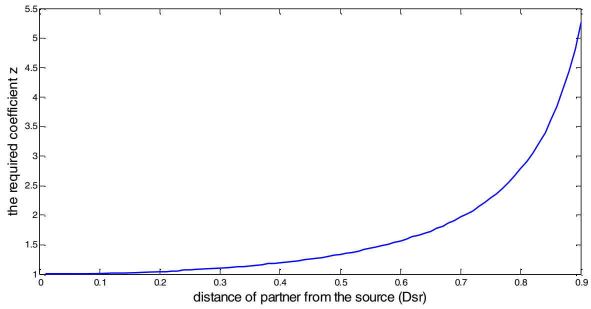

Figure 1. Required amount of z_i for equal SNR (k=1)

Now, we can use the estimation of PDF to estimate the outage probability.

$$P_{out} = \mathrm{Prob}\{I < R\} = \mathrm{Prob}\left\{\sum_{i=0}^{m}\Lambda_i < \left(2^{(m+1)R}-1\right)\right\}$$

$$= \frac{\left(2^{(m+1)R}-1\right)^{m+1}}{(m+1)!} * \frac{1}{\prod_{i=0}^{m}\lambda_i} + O\left(\frac{\left(2^{(m+1)R}-1\right)^{m+1}}{(m+1)!} * \frac{1}{\prod_{i=0}^{m}\lambda_i}\right) \tag{5}$$

Where the last term is obtained from Taylor series expansion that can be derived by the Moment Generating Function technique.

By using (3) and (5), we will reach to the approximated value of outage probability. This value is equal to the used approximated values of [5] and [6], which are used for optimal power allocation.

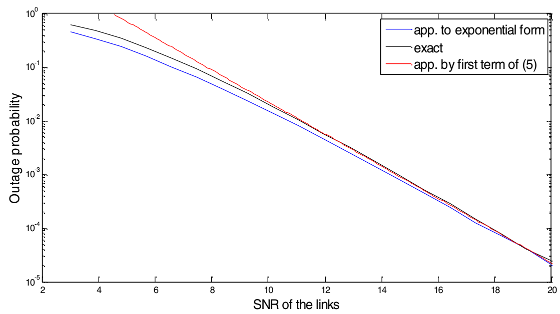

Figure 2. Outage probability (exact and approximately)

For the scenario with one source and destination, two partners with locations $[D_{sr_1}, D_{r_1 d}] = [0.36, 0.72]$ and $[D_{sr_2}, D_{r_2 d}] = [0.71, 0.32]$ and for equal power allocation strategy, we have plotted the exact outage probability (using channel realization), outage probability (using PDF approximation) and approximated value of (5) in figure 2. This figure shows that both approximations have an acceptable accuracy.

IV. Optimal Power Allocation

In the previous section, we modeled the behavior of the outage probability with respect to partner's SNR. In this section, we present the problem of minimizing power with outage probability constraint for a given set of partners. This means that by solution of this section, we can determine the minimum required power for satisfying the target outage probability if all partners in the set are active and based on this, the partner selection algorithm for finding the best partners for minimization of the required power is presented in the next section. The optimal power allocation with outage probability constraint can be represented as follows:

$$min\, P_s + \sum_{i=1}^{m} P_{r_i} \tag{6}$$

$$subject\ to\ P_{out}\left(P_s + \sum_{i=1}^{m} P_{r_i}\right) \le P_{out-th} \tag{6-1}$$

$$P_{max} \ge P_s\, , P_{r_i} \ge 0 \tag{6-2}$$

By approximation (5), the first constraint can be replaced by (6-1').

$$\prod_{i=0}^{m} \lambda_i \ge \frac{\left(2^{(m+1)R}-1\right)^{m+1}}{(m+1)!P_{out-th}} \tag{6-1'}$$

The second constraint is actually 2(m+1) constraints. We assume that all partners in the set are active, which means that all constraint of $P_{r_i} \ge 0$ are inactive in the Karush-Kuhn-Tucker (KKT) method. This is because we try to solve the optimal power allocation with a fixed set of partners and by activating each constraint of $P_{r_i} \ge 0$, the partners set is changed. If we use the formulation of the previous section, (6) can be represented by (7).

$$min\, P_T = \lambda_0 D_{sd}^{\alpha} + \sum_{i=1}^{m}\left(\lambda_0 D_{r_i d}^{\alpha} z_i\left(\frac{\lambda_i}{\lambda_0}, D_{sr_i}\right)\right) \tag{7}$$

$$s.t.\ \lambda_0{}^{m+1} \prod_{i=1}^{m} \frac{\lambda_i}{\lambda_0} \ge \frac{\left(2^{(m+1)R}-1\right)^{m+1}}{(m+1)!P_{out-th}} \tag{7-1}$$

$$P_{max} \ge P_s\, , P_{r_i} \ge 0 \tag{7-2}$$

Now, the problem is represented by functions of $\frac{\lambda_i}{\lambda_0}$ and λ_0. If we use the KKT method to solve this constrained optimization problem (according to [16]), the solution has the following form:

$$\left(\frac{\lambda_i}{\lambda_0}\right)_{opt} = \frac{D_{sr_i}{}^{\alpha} + \frac{1}{2}\zeta_i D_{r_i d}{}^{\alpha} - \sqrt{\frac{1}{4}\zeta_i{}^2 D_{r_i d}{}^{2\alpha} + (1+\zeta_i) D_{sr_i}{}^{\alpha} D_{r_i d}{}^{\alpha}}}{D_{sr_i}{}^{2\alpha} - D_{sr_i}{}^{\alpha} D_{r_i d}{}^{\alpha}} \tag{8-1}$$

$$\lambda_0 = \frac{\left(2^{(m+1)R}-1\right)^{m+1}}{\sqrt[m+1]{\prod_{i=1}^{m} \frac{\lambda_i}{\lambda_0}(m+1)!P_{out-th}}} \tag{8-2}$$

$$\zeta_i = 1 + \sum_{j=1}^{m}\left(\frac{\lambda_j}{\lambda_0} D_{sr_j}^{\alpha}\right) - \frac{\lambda_i}{\lambda_0} D_{sr_i}^{\alpha} \tag{8-3}$$

If the required power for each partner is greater than P_{max}, according to KKT solution, we must decrease the required $\frac{\lambda_i}{\lambda_0}$ to limit the power to P_{max}. We can compute the required power of each partner and the total required power by (8-1) and (8-2).

We note that the existence of ζ_i change the solution of power allocation to an iterative solution. In this paper we neglect the impact of ζ_i by setting $\zeta_i = 1$. We will show that the performance of this algorithm is very close to iterative algorithm.

In figure 3, the optimal required ratio of the partner SNR to source SNR ($\left(\frac{\lambda_i}{\lambda_0}\right)_{opt}$) for each node in the neighborhood of the source and destination is shown (for $\alpha = 2$). Based on this, the normalized required power for the set which is composed of each node at those points and the source is shown in figure 4. This figure shows that the partners that are located very close to destination are the best nodes with smallest required power. In this figures, we use zero as minimum transmit power and if we use the positive constant for this threshold, the required power for the partners in neighborhood of destination will be increased. In this figures, we ignore the maximum power constraint for comparing between the nodes, too.

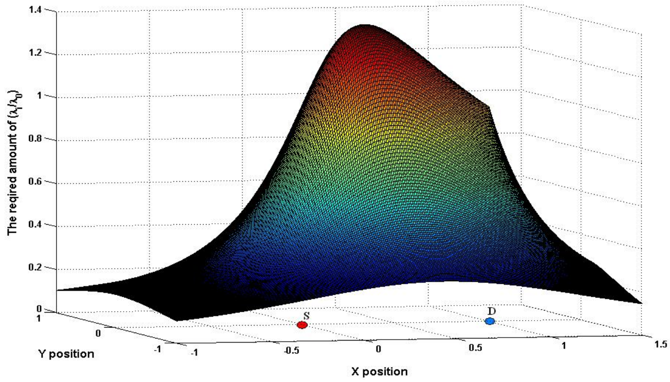

Figure 3. Optimal required $\left(\frac{\lambda_i}{\lambda_0}\right)_{opt}$ for each node

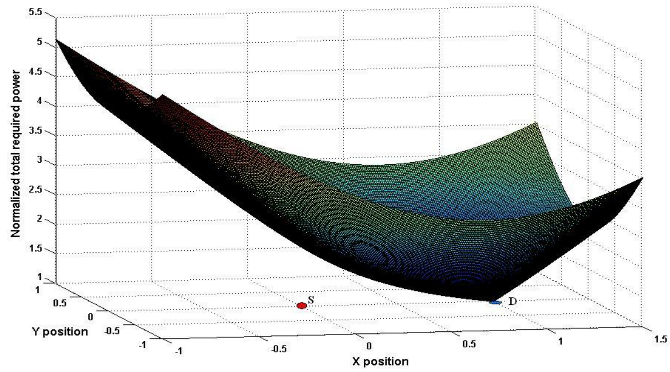

Figure 4. Total required power for the set of node and destination (m=2)

V. Optimal Partner Selection

In the previous section, we show that for each given set of partners, the minimum required power for satisfying the outage probability constraint can be computed simply by (8-1) and (8-2). We must find the set with minimum required power for optimal partner selection. By other meaning, we found the local minimums in the previous section and we will try to find the global minimum between them in this section. The best set with the minimum power can be found by exhaustive search between entire possible sets. Increasing the number of candidate partners will grow the complexity of this search, exponentially. In this section, we present a very low complexity algorithm for finding the best partner set. If the number of candidate partners is N, this algorithm is divided to N steps where in each step, the number of partners are fixed to m (from 1 to N) and the best set with m partners is found with a simple method which is presented below. We can stop this search before $m = N$ and choose the best set. Because increasing m after m_0 (where m_0 is a function of P_{out-th} and condition of the partners) does not reduce the total required power for the best set.

Now, we present the method of finding the best set (minimum power set) for a fixed number of partners (m). First, the metric (9) is computed for all candidate partners and the node is ranked and selected based on this metric.

$$m_1 = \frac{1}{\left(\frac{\lambda_i}{\lambda_0}\right)^{m+1}} \left(D_{sd}^{\alpha} + z_i\left(\frac{\lambda_i}{\lambda_0}, D_{sr_i}\right) D_{r_i d}^{\alpha}\right) \quad (9)$$

This selected set may be not the best set with m partners. So we need a process to change the selected set of m nodes to find the best set. For this goal, we can compare the required power of the selected set and another set in which a given node i in the selected set is replaced with another node j from outside the set.

$$m_{1i} + \frac{\sum_{k\in set} z\left(\frac{\lambda_k}{\lambda_0}, D_{sr_k}\right) D_{r_k d}^{\alpha}}{\left(\frac{\lambda_i}{\lambda_0}\right)^{m+1}} \gtrless m_{1j} + \frac{\sum_{k\in set} z\left(\frac{\lambda_k}{\lambda_0}, D_{sr_k}\right) D_{r_k d}^{\alpha}}{\left(\frac{\lambda_j}{\lambda_0}\right)^{m+1}} \quad (10)$$

If $\frac{\lambda_i}{\lambda_0} \geq \frac{\lambda_i}{\lambda_0}$, then the node i can not be replaced by node j in the best set but if $\frac{\lambda_i}{\lambda_0} < \frac{\lambda_i}{\lambda_0}$, then node i may be replaced by node j. So, we can remove the candidate partners which their $\left(\frac{\lambda_i}{\lambda_0}\right)_{opt}$ values are smaller than the minimum value of $\left(\frac{\lambda_i}{\lambda_0}\right)_{opt}$ in the selected partners in first stage from the selection of this step. The inequality (10) shows that we must compute the other metric than (9) for each node but unfortunately this metric change for different partners in the set. By this reason, we replace D_{sd}^{α} term in (9) by C in (11) and make a new metric which equal to minimum required metric for the partners in the set and compute this for the candidate partners.

$$C = D_{sd}^{\alpha} + \sum z\left(\frac{\lambda_k}{\lambda_0}, D_{sr_k}\right) D_{r_k d}^{\alpha} - \max\left(z\left(\frac{\lambda_k}{\lambda_0}, D_{sr_k}\right) D_{r_k d}^{\alpha}\right) \quad (11)$$

Using this new metric, we can choose the new partner set which has a greater chance to be in optimal set because if we write (10) again and divide each side to metric and non-metric terms, the non-metric term has the weaker impact in satisfying the inequality with respect to (10). In this stage, three different cases may happen:

1- The previous set is selected again. Then we conclude that this set is the optimal set.
2- The required power of the new set is less than previous set. In this case, we must re-compute (11) and find the set based on this.
3- The required power of the new set is greater than the previous set. This case happens infrequently and we must decrease (11) and compute the metric for all nodes and find the set based on this.

This will be shown that the number of iterations in each step is very small and about 1 and 2. We note that the first selected set by metric (9) is near optimal set and the power of the set is near the optimal power and the iterative manner only is added to select the optimal set with more accuracy.

VI. SIMULATION RESULTS

In this section, we present the simulation results to show the accuracy of the proposed power allocation and partner selection algorithm. We have implemented a full search program using the technique of numerical optimization of [16] and based on the optimal allocation of power with total power constraint proposed in [6]. This program does an exhaustive search within all possible sets and finds the required power for each set to choose the best set with minimum required power. We compare our results with the results of this full search approach.

As a sample simulation set up, 20 candidate nodes for cooperation in transmission from source "s" to destination "d" are shown in figure 5. We assume that R=1 bit per second per Hertz, $N_0B = 5 \times 10^{-4}W$ and $d_{sd} = 1m$ (which means that $D_{r_i d} \equiv d_{r_i d}$ and $D_{sr_i} \equiv d_{sr_i}$)

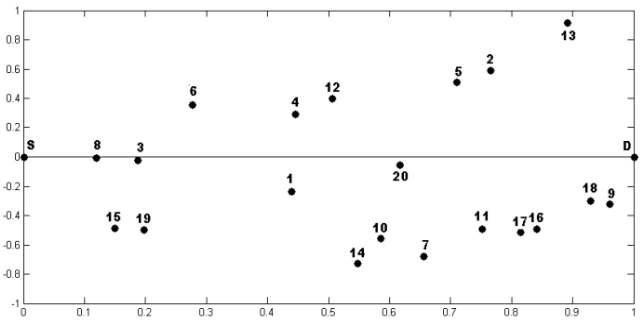

Figure 5. Location of 20 candidate partners

In table I, the required total powers for transmission with different target outage probability for the set of {n_{20}, n_{18}, n_9} partners are shown. The small differences between required power of our algorithm and optimal results show the accuracy of our optimal power allocation algorithm. In the fourth column of this table, the percentage of difference of exact outage probability of our power allocation (in third row) and the target outage probability is shown. This difference is the result of difference between exact outage probability and approximation (5). For some positions the approximation (5) is under provisioning. This behavior can be corrected if we use approximation (5) by a coefficient greater than 1. For example in this table, the prospect required power will be increased with amount of 0.19dBm by using appropriate coefficient. The adding of this coefficient increases the prospect total power of our algorithm, but ensures satisfying the outage probability constraint.

TABLE I. REQUIRED POWER FOR THE SET {N20 ,N18 ,N9}

P_{out-th}	Power dBm(full search)	Power dBm(our alg.)	Difference of outage of our result with P_{out-th} (%)
0.01	13.29	13.6	2%
0.0014	15.46	15.74	14%
1.18E-04	18.31	18.42	20%
1.00E-05	20.99	21.1	5%

In table II, assuming P_{out-th}=1.4e-3, the best sets and required powers with different m values are shown. It is seen that our proposed algorithm of partner selection provides quite accurate results. In fourth column of this table, the number of iteration of our algorithm for finding the best set is shown. It can be seen that in most cases, one round of calculations is sufficient and the largest number of iterations is 2. This demonstrates the simplicity and low complexity of our proposed partner selection algorithm.

TABLE II. THE BEST SETS AND REQUIRED POWERS

m	Optimal set (full search)	Optimal set (our alg.)	Number of iter.
1	{n20} P_T=15.16	{n20} P_T=15.44	1
2	{n20,n18} P_T=14.53	{n20,n18} P_T=14.64	1
3	{n20,n18,n9} P_T=15.46	{n20,n18,n9} P_T=15.74	1
4	{n20,n18,n9,n1} P_T=17.32	{n20,n18,n9,n1} P_T=17.77	2

In figure 6, the required power for the best set of our algorithm is compared with the optimal results for the best set obtained by full search with different outage probability. It can be seen that results of our algorithm are very close to the optimal results. In this figure, if $P_{out-th} \in [1e-5\,, 7.2e-5]$, the best set is $\{n_{20}, n_{18}, n_9\}$, if $P_{out-th} \in [7.2e-5\,, 6.1e-3]$, the best set is $\{n_{20}, n_{18}\}$ and if $P_{out-th} \in [6.1e-3\,, 1e-2]$, the best set is $\{n_{20}\}$. This shows that if the target outage probability is decreased, then a higher number of partners are required to increase the order of diversity and satisfy the required transmission conditions.

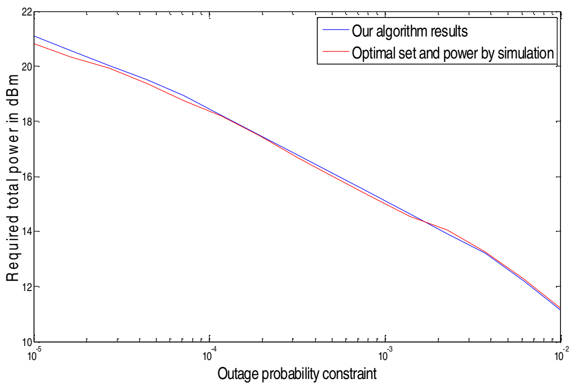

Figure 6. The required power for the best set

VII. CONCLUSION

In this paper we presented an algorithm to find the best set of partners with minimum required power among all candidate partners given a target outage probability in Amplify and Forward Cooperative Diversity. We demonstrated that the algorithm can converge rapidly to the desired set of partners and the results are very close to the optimal results obtained by an exhaustive full search method.

ACKNOWLEDGMENT

Authors wish to thank Sina Khaleghi for his helpful comments on the paper. Thanks are also extended to the Iran Telecommunication Research Center (ITRC) for supporting this work.

REFRENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity– part I: system description,” IEEE Trans. Commun., vol. 51, no. 11, pp.1927–1938, Nov. 2003.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” IEEE Trans Inform. Theory, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[3] J. N. Laneman and G. W. Wornell, “Distributed space–time coded protocols for exploiting cooperative diversity in wireless networks,” IEEE Trans. Inf. Theory, vol. 49, no. 10, pp. 2415–2525, Oct. 2003.

[4] J. Luo, R. S. Blum., L. J. Cimini, L. J. Greenstein and A. M. Haimovich, “Decode-and-Forward Cooperative Diversity with Power Allocation in-Wireless Networks,” IEEE Trans. on Wireless Communications, vol. 6, no. 3, pp. 793-799, March 2007.

[5] R. Annavajjala, P. C. Cosman, L. B. Milstein, “Statistical channel knowledge-based optimum power allocation for relaying protocols in the high SNR regime,” IEEE Journal on Selected Areas in Communications, vol. 25, no. 2, pp.292–305, Feb. 2007.

[6] Y. Zhao, R. Adve, and T. Lim, “Improving amplify-and-forward relay networks: optimal power allocation versus selection,” IEEE Trans. on Wireless Communications, vol. 6, no. 8, pp. 3114-3123, August 2007.

[7] A. Bletsas, A. Khisti, D.P. Reed, A. Lippman, “A Simple Cooperative. Diversity Method based on Network Path Selection”, IEEE Journal on Selected Areas in Com., vol. 24, no. 3, pp.659–672, Mar. 2006.

[8] A. Nosratinia, T. E. Hunter, “Grouping and partner selection in cooperative wireless networks,” IEEE Journal on Selected Areas in Communications, vol. 25, no. 2, pp. 369-378, Feb. 2007.

[9] N. Ahmed, M.A. Khojastepour, B. Aazhang, “Outage Minimization and Optimal Power Control for the Fading Relay Channel,” IEEE Information Theory Workshop, San Antonio, TX, October 24-29, 2004.

[10] C. Fischione, A. Bonivento, A. Sangiovanni-Vincentelli and K. H. Johansson, “Cooperative Diversity with Disconnection Constraints and Sleep Discipline for Power Control in Wireless Sensor Networks”, in Proc. of IEEE 64th Semiannual Vehicular Technology Conference - Spring 2006 (IEEE VTC Spring 06), Melbourne, Australia, May 2006.

[11] T. Himsoon, W.P. Siriwongpairat, Z. Han, and K.J.R. Liu, "Lifetime Maximization via Cooperative Nodes and Relay Deployment in Wireless Networks", IEEE Journal of Selected Areas in Communications, Special Issue on Cooperative Communications and Networking, vol 25, no 2, pp.306-317, Feb 2007.

[12] V. Mahinthan, L. Cai, J. W. Mark and X. Shen, “Maximizing Cooperative Diversity Energy Gain for Wireless Networks,” IEEE Trans. Wireless Commun., vol. 6, no. 7, pp.2530–2539, Jul. 2007.

[13] E. Yazdian, M.R. Pakravan, “Adaptive Modulation Technique for Cooperative Diversity in Wireless Fading Channel,” PIMRC, Sep 2006.

[14] D. S. Michalopoulos, G. K. Karagiannidis, T. A. Tsiftsis and R. K. Mallik, " Distributed Transmit Antenna Selection (DTAS) under Performance or Energy Consumption Constraints", IEEE Transactions on Wireless Communications, Vol. 7, No. 4, 1168-1173, April 2008.

[15] A.Papoulis, S. U. Pillai, “Probability, random variables and stochastic processes,” 4th edition, Mc Graw Hill publication.

[16] J.Nocedal and S. J. Wright, “Numerical optimization,” Springer press.