Balanced Truncation with Spectral Shaping for RLC Interconnects

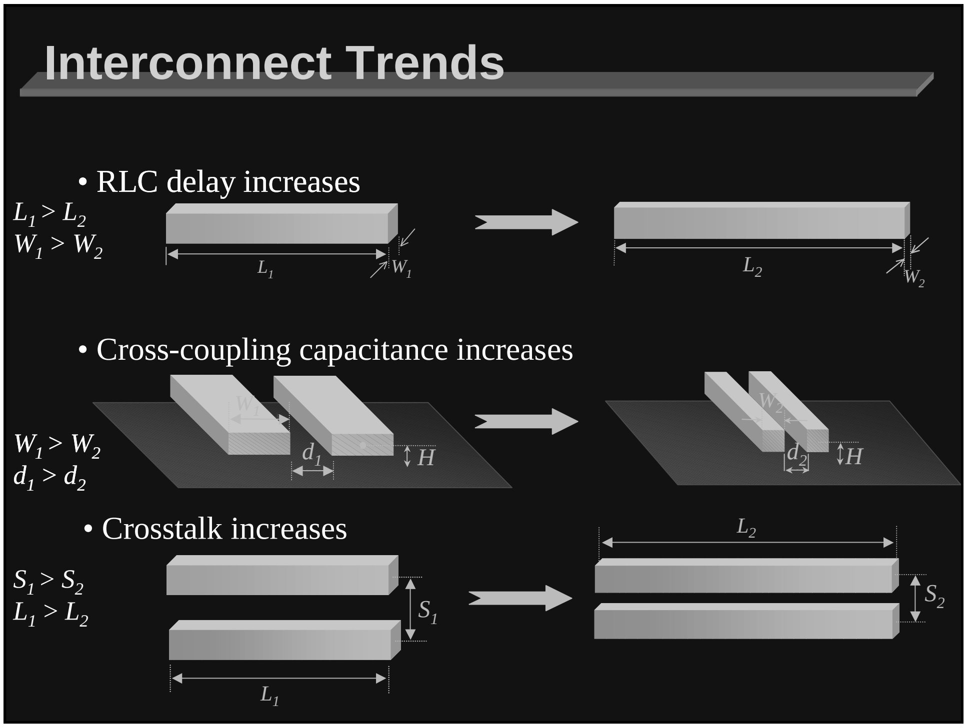
Payam Heydari and Massoud Pedram

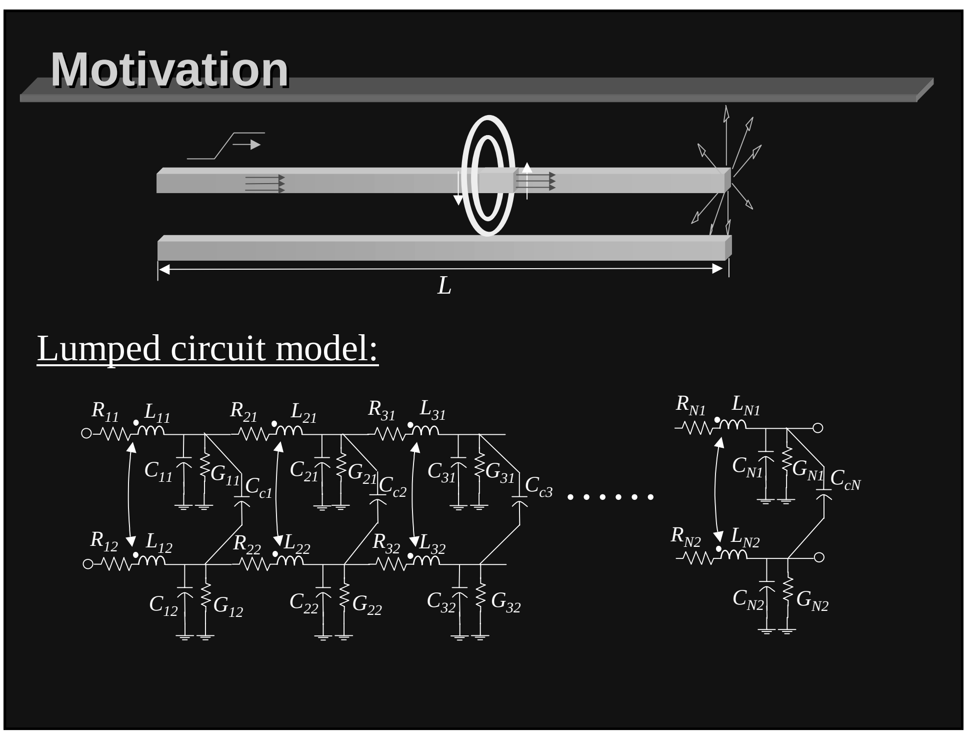
Department of Electrical Engineering-Systems
University of Southern California
Los Angeles, CA 90089, U.S.A.

ASP-DAC 2001, Yokohama, JAPAN

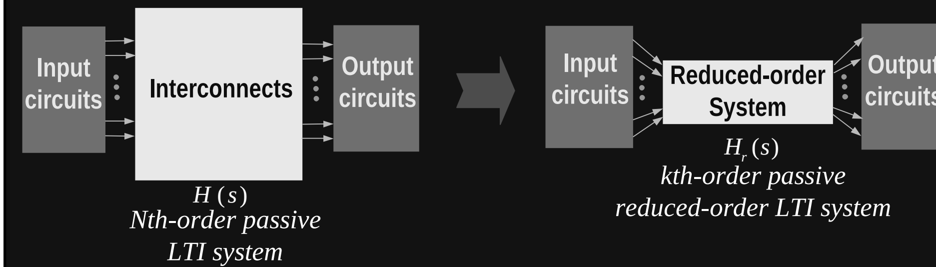
Outline

- Interconnect Trends
- Motivation and Problem Definition
- Prior Work
- Balanced Truncation
- Frequency-weighted Balanced Truncation
- Experimental Results
- Conclusions

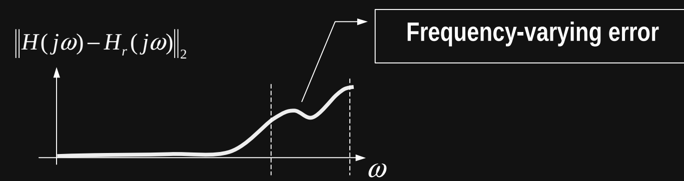

Interconnect Trends
• RLC delay increases
$L_1 > L_2$
$W_1 > W_2$
L_1
W_1
L_2
W_2
• Cross-coupling capacitance increases
$W_1 > W_2$
$d_1 > d_2$
d_1
H
d_2
H
• Crosstalk increases
$S_1 > S_2$
$L_1 > L_2$
S_1
L_1
L_2
S_2


Motivation
L
Lumped circuit model:
R_{11} L_{11} R_{21} L_{21} R_{31} L_{31} R_{N1} L_{N1}
C_{11} G_{11} C_{c1} C_{21} G_{21} C_{c2} C_{31} G_{31} C_{c3} …… C_{N1} G_{N1} C_{cN}
R_{12} L_{12} R_{22} L_{22} R_{32} L_{32} R_{N2} L_{N2}
C_{12} G_{12} C_{22} G_{22} C_{32} G_{32} C_{N2} G_{N2}

Problem Definition



- Need to find a methodology for analyzing a computationally intensive system of tightly coupled interconnects



- Need to spectrally reshape the error

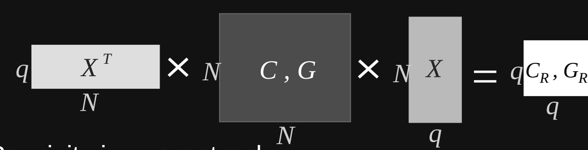
Prior Work

- Explicit moment matching
  - **A**symptotic **W**aveform **E**valuation (Pillage *et al.*, TCAD'90)
    - ➢ Write the Taylor series expansion of the original system transfer function
    - ➢ Truncate the Taylor series expansion to the (*2q-1*)st-order polynomial
    - ➢ Given the truncated polynomial, use Pade approximation to find a qth-order rational function
  - ☹ Numerical instability
  - ☹ Unstable poles even for a low-order moment matching

- Krylov-subpace-based model-order reduction
  - **P**ade **V**ia **L**anczos (Feldmann *et al.*, TCAD'95) and Arnoldi (Silveira *et al.*, ICCAD'96)
    - ➢ Construct a set of orthonormal basis that spans the Krylov-subpace
    - ➢ Using the Lanczos or Arnoldi method, find a projection matrix that transforms the system matrix to a lower order matrix in the new space
    - ➢ For the Arnoldi method, the projected matrix is an upper Hessenberg matrix
  - ☹ Passivity is not guaranteed

Prior Work

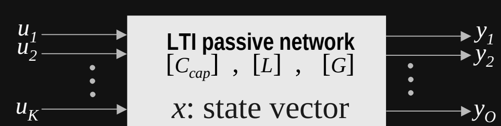
- Passivity guaranteed model-reduction (Odabasioglu *et al.*, TCAD'98, Kerns *et al.*, DAC'97)
  - Obtain a congruence transformation using the Arnoldi method
  - Apply the congruence transformation directly to the element matrices



  - ☺ Passivity is guaranteed
  - ☹ Like other Krylov-subspace-based methods, does not provide a provable error bound for the reduced system

- Balanced Truncation (Silveira *et al.*, TCHMT'94, Rabiei *et al.*, ASP-DAC'99)
  - ☺ There is a provable error bound for the reduced system
  - ☹ More computational complexity compared to Krylov-subspace methods
  - ☼ Would be great if the error can be reshaped in the frequency domain

Balanced Truncation



$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

$$A = L^{-1}G \qquad \mathbf{L} = \begin{bmatrix} C_{cap} & 0 \\ 0 & L \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} G & E \\ -E^T & 0 \end{bmatrix}$$

- For any passive LTI system, there exist symmetric, positive-definite matrices, *P* and *Q*, that satisfy the Lyapunov equations:

$$AP + PA^T + BB^T = 0 \quad \text{and} \quad A^TQ + QA + C^TC = 0$$

Physical interpretation of the controllability grammian:

For all possible inputs to the system that are able to transfer the state from initial state x_0 to the zero state, the input with the minimum energy is related to the controllability grammian

Balanced Truncation

Definition:

The Hankel singular values of the system transfer function $H(s)=C(sI - A)^{-1}B$, are the square-roots of the eigenvalues of PQ

Definition:

An LTI system is called *balanced* if $P=Q$

Importance of Hankel singular values:

- In a balanced system the value of i-th Hankel singular value, σ_i, is associated with i-th state variable, x_i $(1 \le i \le n)$
- σ_i is a relative measure of contribution that x_i makes to the input-output behavior $(1 \le i \le n)$

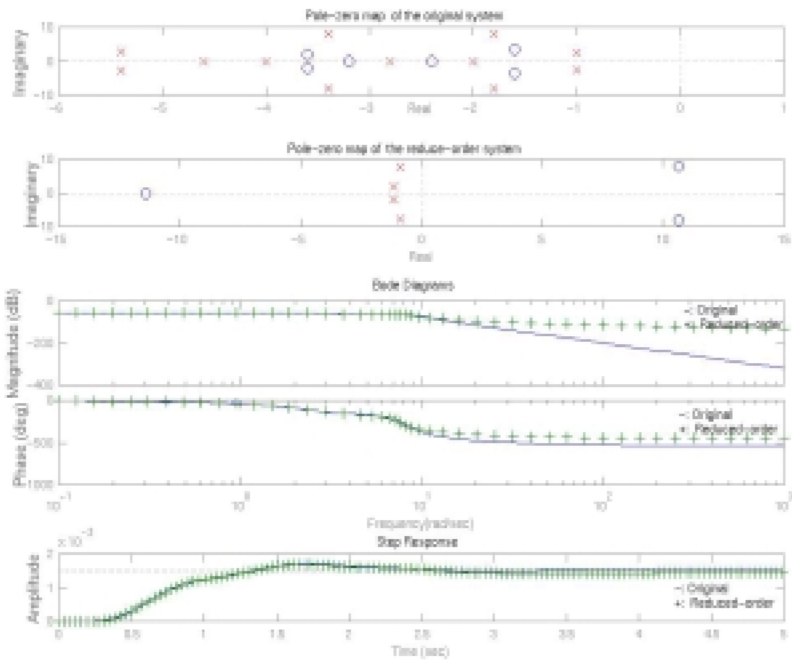
Balanced Truncation

- Apply the Cholesky factorization on matrix Q: $Q = R^T R$
- Diagonalize the matrix RPR^T: $RPR^T = U\Sigma^2 U^T$ with $U^T U = I$
- Construct the balancing transformation: $T = \Sigma^{-1/2} U^T R$
- Using T, obtain the new coordinate transformed balanced system: $A_r = TAT^{-1} \quad B_r = TB \quad C_r = CT^{-1}$
- The kth-order truncated balanced realization is:

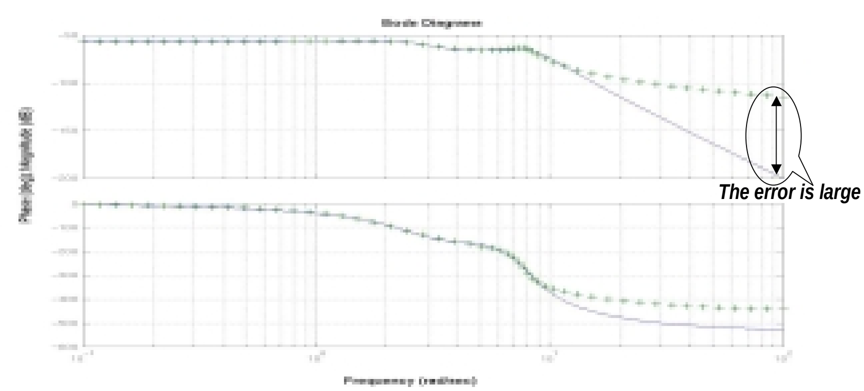
$$A_r = \begin{bmatrix} A_{r,11} & A_{r,12} \\ A_{r,21} & A_{r,22} \end{bmatrix}_{k \times k} \qquad B_r = \begin{bmatrix} B_{r,1} \\ B_{r,2} \end{bmatrix}_{k \times p} \qquad C_r = \begin{bmatrix} C_{r,1} & C_{r,2} \end{bmatrix}_{q \times k}$$

- The reduced-order model is stable and the L^∞-error is bounded:

$$\left\| H(s) - H_r^k(s) \right\|_\infty \le 2 \sum_{i=k+1}^{n} \sigma_i$$



Frequency-weighted Balanced Truncation

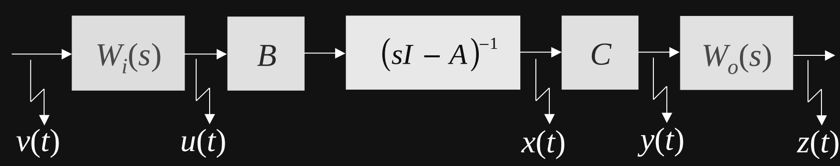


- Need to minimize the error in the frequency range of interest

Frequency-weighted Balanced Truncation

- The frequency-weighted balanced realization problem is to calculate $H_r^k(s)$ of degree *k* (*k*<*n*) so as to minimize

$$\left\| W_o(s)\left(H(s) - H_r^k(s)\right)W_i(s) \right\|_\infty$$



? What set of points in the x-state space is a part of zero initial condition response for some bounded and weighted input, *v*(*t*)?

? What set of points in the x-state space as initial conditions produce a bounded and weighted output, *z*(*t*)?

Frequency-weighted Balanced Truncation

$$W_i(s) = C_i\left(sI - A_i\right)^{-1}B_i + D_i$$
$$W_o(s) = C_o\left(sI - A_o\right)^{-1}B_o + D_o$$

- The controllable set of the system, $H(s)W_i(s)$, is the answer to the first question

<u>Controller-form realization</u>

$$\overline{A_i} = \begin{bmatrix} A & B_i C \\ 0 & A_i \end{bmatrix} \qquad \overline{B_i} = \begin{bmatrix} BD_i \\ B_i \end{bmatrix} \qquad \overline{C_i} = \begin{bmatrix} C & 0 \end{bmatrix}$$

- The observable set of the system, $W_o(s)H(s)$, is the answer to the second question

<u>Observable-form realization</u>

$$\overline{A_o} = \begin{bmatrix} A & 0 \\ B_o C & A_o \end{bmatrix} \qquad \overline{B_o} = \begin{bmatrix} C \\ 0 \end{bmatrix} \qquad \overline{C_o} = \begin{bmatrix} D_o C & C_o \end{bmatrix}$$

Frequency-weighted Balanced Truncation

Lyapunov equation

Controller-form realization

$$\overline{A_i} = \begin{bmatrix} A & B_iC \\ 0 & A_i \end{bmatrix} \quad \overline{B_i} = \begin{bmatrix} BD_i \\ B_i \end{bmatrix} \quad \overline{C_i} = \begin{bmatrix} C & 0 \end{bmatrix}$$

$\overline{P}$: **Controllability grammian** $= \begin{bmatrix} P & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$

Lyapunov equation

Observer-form realization

$$\overline{A_o} = \begin{bmatrix} A & 0 \\ B_oC & A_o \end{bmatrix} \quad \overline{B_o} = \begin{bmatrix} C \\ 0 \end{bmatrix} \quad \overline{C_o} = \begin{bmatrix} D_oC & C_o \end{bmatrix}$$

$\overline{Q}$: **Observability grammian** $= \begin{bmatrix} Q & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$

Frequency-weighted Balanced Truncation

- Expand the $n \times n$ upper left corner of the Lyapunov equations

$$AP + PA^T + \underbrace{BC_iP_{12} + P_{12}^T C_i^T B^T + BD_iD_i^T B}_{X} = 0$$

$$A^TQ + QA + \underbrace{Q_{12}B_oC + C^TB_o^TQ_{12}^T + C^TD_o^T D_oC}_{Y} = 0$$

$X = X^T$

$Y = Y^T$

⇒

$\exists U, S \in \Re^{n\times n} : U^TU = I$, $S : Diag(\cdot)$ *such that* $X = USU^T$

$\exists V, Z \in \Re^{n\times n} : V^TV = I$, $Z : Diag(\cdot)$ *such that* $Y = VZV^T$

Frequency-weighted Balanced Truncation

$$\begin{bmatrix} X \\ Y \end{bmatrix} \Rightarrow \begin{bmatrix} S = diag\,(s_1, s_2, ..., s_n) \\ Z = diag\,(z_1, z_2, ..., z_n) \end{bmatrix}$$

- Assume that $rank\,(X) = i\,,\; i \le n$ and $rank\,(Y) = j\,,\; j \le n$

$$\overline{B} = U diag\,(|s_1|^{1/2}, |s_2|^{1/2}, ..., |s_i|^{1/2}, 0, ...,0)$$
$$\overline{C} = diag\,(|z_1|^{1/2}, |z_2|^{1/2}, ..., |z_j|^{1/2}, 0, ...,0) V^T$$

$$A\hat{P} + \hat{P}A^T + \overline{B}\,\overline{B}^T = 0$$
$$A^T\hat{Q} + \hat{Q}A + \overline{C}^T\,\overline{C} = 0$$

- Let $\hat{P}$ and $\hat{Q}$ denote the solutions of the Lyapunov equations
- Repeat the same steps that are used for a unity-weighted system

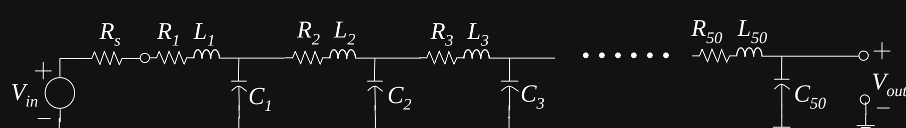
Experimental Results

<u>A single lossy transmission line</u>

Modeled by a ladder of 50 RLC lumped sections

Values of the components are normalized

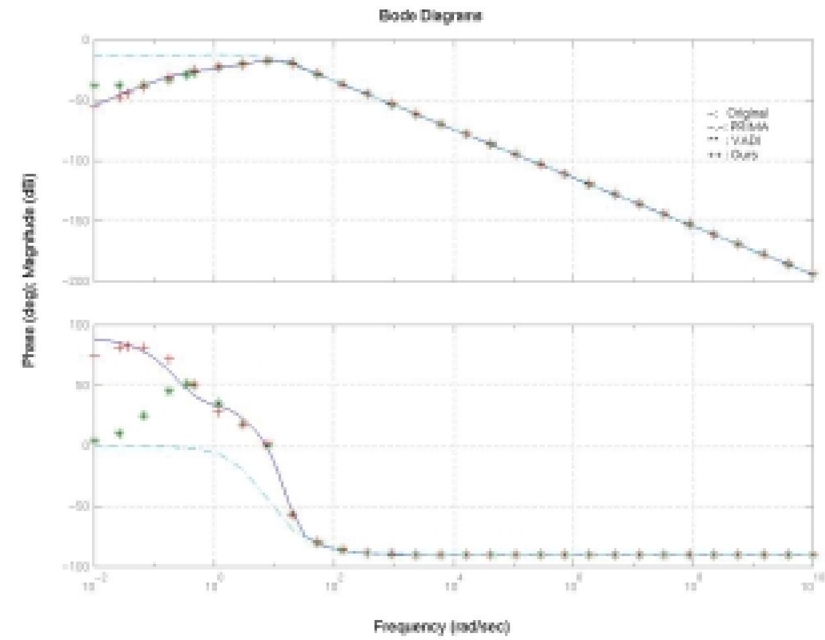
$$W_i(s) = \frac{1}{s + 0.4}$$



$$C_1 = C_2 = ... = C_{50} = 2\times10^{-2}$$
$$L_1 = L_2 = ... = L_{50} = 0.5$$
$$R_1 = R_2 = ... = R_{50} = 0.25$$
$$R_s = 0$$

Order of the system: 100

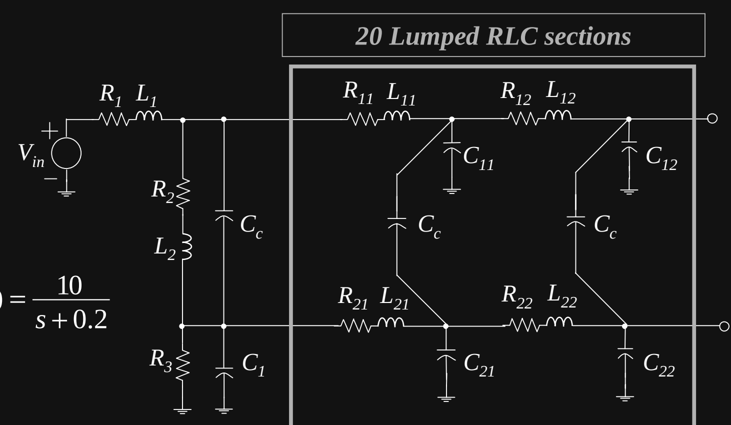
Order of the reduced system: 5



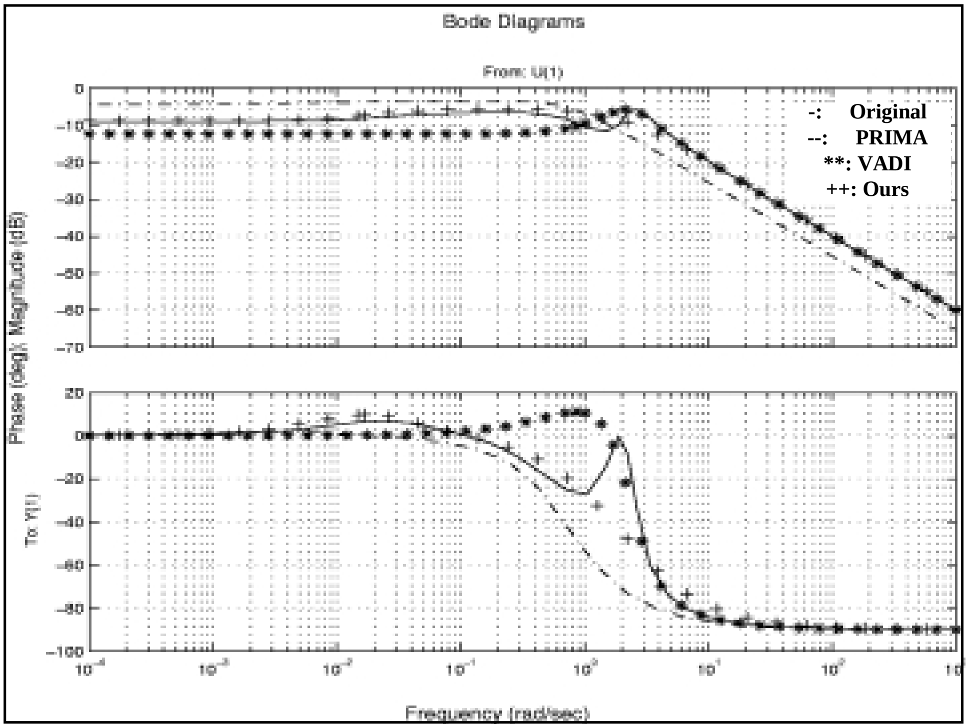
Experimental Results

<u>Two capacitively coupled lossy transmission lines</u>

Each line is modeled by a ladder of 20 RLC lumped sections
Values of the components are normalized



$$W_o(s) = \frac{10}{s + 0.2}$$



Conclusions

- A frequency-weighted balanced truncation technique for model-reduction of multiport RLC interconnect was proposed
- This technique yields passive reduced order model even when both input and output weightings are applied
- The Lyapunov equations are efficiently solved by Krylov-subspace-based methods in combination with an iterative Lyapunov equation solver
- Experimental results and comparison with truncated balanced realization techniques and PRIMA show the higher accuracy of our approach